helicoid vane; R_2, radius of the inner surface of the outer tube; α, central angle; A, a constant; B, activation energy for the flow process; R, gas constant; T, absolute temperature; $\chi = (\chi_1, \chi_2)$, point in two-dimensional Euclidean space; h_β, spacing of computing net ω_h; ω_h, set of interior nodes; γ_h, set of boundary nodes; G, a domain; $\overline{G}$, a domain with boundary Γ; $L_\beta u$, Laplace operator; C_β, straight line through an interior node; Λ, difference operator; τ^*, z-spacing of net; h^*_β, distance from irregular node χ to boundary node $\chi^{(-1\beta)}$ or $\chi^{(+1\beta)}$; $h^*_{i\beta}$, distance from nodes $\omega_{h,\beta}$ next to the boundary to boundary nodes $h_{h,\beta}$; $N'_{i\beta}$, indices of left boundary nodes in a matrix in the direction of χ_β; $N''_{i\beta}$, indices of right boundary nodes; $A_{i\beta}$, $B_{i\beta}$, $C_{i\beta}$, sweep coefficients in difference equation; Q, volumetric flow rate; θ, dimensionless temperature; $\bar{v}$, average flow velocity; Z, duct length; l, length of the initial thermal section; F, useful cross section of duct; $\bar{v}_{\chi_1}$, $\bar{v}_z$, $\bar{\alpha}$, $\bar{F}$, discrete values of functions.

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TRANSIENT PROCESSES IN SHEAR FLOWS OF A VISCOELASTIC FLUID.

III. ELASTIC RECOVERY*

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A mathematical description of the elastic recovery effect associated with viscoelastic shear flow is given.

In this article we investigate the transient process associated with the shear flow of a viscoelastic fluid in the clearance space between coaxial cylinders (problem 3) [1] when the outer cylinder is rigidly fixed and the inner one set in motion by the action of a constant external torque. The applied torque acts for a period of time t*, after which it is removed. The detailed mathematical statement of the problem and a procedure for its numerical solution are described in [1].

The external torque drives the cylinder and the fluid. As in problem 2, after several transits of a shear wave across the clearance space a quasisteady fluid flow regime is established, for which the conditions of realization are described in [2]. We now give a qualitative analysis of the influence of the rheological properties of the fluid on the laws

*The problem treated in Parts I and II [1, 2] and the present article are discussed in application to the Bird—Carreau, Meister, and MacDonald—Bird—Carreau nonlinear models in a paper by Z. P. Shul'man, S. M. Aleinikov, and B. M. Khusid, Rheodynamics and Heat Transfer in Unsteady Shear Flows of Nonlinear Hereditary Media, Preprint No. 6 of the Institute of Heat and Mass Transfer of the Academy of Sciences of the Belorussian SSR [in Russian], ITMO AN BSSR, Minsk (1982).

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governing the rotation of the inner cylinder in a viscoelastic fluid in the quasisteady stage, neglecting the inertia of the fluid (see [2] for more details). For the stated problem we arrive at the system of equations

$$I\ddot{\varphi} - 2\pi R_I^2 L(\tau_I + \tau_{II}) = M,$$
$$\tau_I + \lambda \frac{d\tau_I}{dt} = -\eta_1\dot{\varphi}/\delta_1, \ \tau_{II} = -\eta_2\dot{\varphi}/\delta_1, \tag{1}$$
$$\varphi(0) = \dot{\varphi}(0) = \tau_I(0) = 0.$$

Under the adopted initial conditions it is assumed that the angle of rotation and angular velocity acquired by the cylinder in the "rise time" leading up to the quasisteady fluid flow regime are insignificant, as are the stresses in a Maxwellian element. The system (1) and its characteristic equation take the following form in our previous notation convention [2]:

$$\dot{\Omega} + 2\beta\varepsilon\Omega - 2(1-\beta)\varepsilon\bar{\tau}_I = 0, \ \bar{\tau}_I + \lambda\frac{d\bar{\tau}_I}{dt} = -\Omega, \tag{2}$$

$$\Omega(0) = -\Omega_{st}, \ \bar{\tau}_I(0) = \Omega_{st}, \ k^2 + \left(2\beta\varepsilon + \frac{1}{\lambda}\right)k + \frac{2\varepsilon}{\lambda} = 0, \tag{3}$$

where $\dot{\varphi} = \Omega_{st} + \Omega$. In the case of rotation of the cylinder in a viscous fluid ($\lambda = 0$)

$$\Omega = -\Omega_{st}\exp(-2\varepsilon t). \tag{4}$$

This solution shows that in a viscous fluid the velocity of rotation of the inner cylinder increases from zero to the steady value Ω_{st} in a time $1/2\varepsilon$ (Fig. 1a). For a viscoelastic fluid the nature of the rotation of the inner cylinder depends on the relation between the relaxation time λ and the quantity $1/\varepsilon$. Next we give a quantitative analysis of various limiting cases.

When the relaxation time λ is much smaller than the characteristic rise time to a steady value of the velocity of rotation of the cylinder in a viscous fluid ($\varepsilon\lambda \ll 1$), the solution of the system (2) coincides with (4). The elastic properties of the fluid do not affect the rotation of the cylinder.

When the relaxation time λ is much greater than the rise time to steady rotation of the cylinder in a viscous fluid ($\varepsilon\lambda \gg 1$), the solution of the system (2) takes the form

$$\Omega = -\Omega_{st}\exp(-t/\beta\lambda) + \frac{\Omega_{st}}{\beta}[\exp(-t/\beta\lambda) - \exp(-2\beta\varepsilon t)]. \tag{5}$$

The variation of Ω for $\lambda \gtrsim t \gtrsim 1/\varepsilon$ corresponds to the case in which the influence of inertial forces in the equation of motion of the cylinder is slight ($\dot{\Omega} \approx 0$). The stresses in a Maxwellian element vary according to the law $\bar{\tau}_I \approx \beta\Omega/(1-\beta)$, and the rheological relation yields $\Omega \sim \exp(-t/\lambda\beta)$. It is evident from the solution (5) that the curve of $\dot{\varphi}(t)$ reaches a maximum at $t \sim 1/2\beta\varepsilon$ (Fig. 1b), after which the velocity of rotation of the inner cylinder decreases monotonically to the value Ω_{st} according to the law

$$\dot{\varphi} \approx \Omega_{st}\left[1 + \left(\frac{1}{\beta} - 1\right)\exp(-t/\lambda\beta)\right].$$

For $\varepsilon\lambda = (1 \pm \sqrt{1-\beta})^2/2\beta^2$ the characteristic equation (3) has multiple roots. This situation corresponds to the case in which the viscous friction and elastic forces of the fluid become commensurate. For $(1 - \sqrt{1-\beta})^2/2\beta^2 < \varepsilon\lambda < (1 + \sqrt{1-\beta})^2/2\beta^2$ the roots of Eq. (3) are complex conjugates of one another. The motion of the inner cylinder has an oscillatory behavior. We note that for rotation of the cylinder in a viscous fluid this situation does not arise.

The solution of the system (2) for $\varepsilon\lambda \sim (1 \pm \sqrt{1-\beta})^2/2\beta^2$ is written in the form

$$\Omega = \Omega_{st}\exp\left(-\frac{1 \pm \sqrt{1-\beta}}{\beta\lambda}t\right)\left(-\cos At \pm \frac{\sqrt{1-\beta}(1 \pm \sqrt{1-\beta})^2}{\beta^2\lambda A}\sin At\right),$$

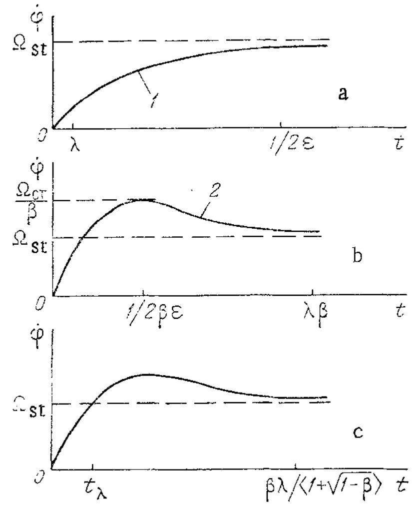

Fig. 1

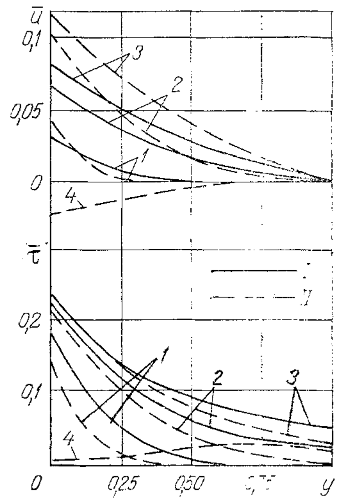

Fig. 2

Fig. 1. Angular velocity of the inner cylinder driven by a constant external torque. a) Viscous and viscoelastic ($\varepsilon\lambda \ll 1$) fluids; b) viscoelastic fluid ($\varepsilon\lambda \gg 1$); c) viscoelastic fluid, $\varepsilon\lambda \sim (1+\sqrt{1-\beta})^2/2\beta^2$ $\left(1 — \Omega_{st}\cdot(1-\exp(-2\varepsilon t));\ 2 — \Omega_{st}\left[1+\left(\frac{1}{\beta}-1\right)\exp(-t/\lambda\beta)\right]\right)$.

Fig. 2. Profiles of the velocity and tengential stresses in a viscous (I) and a viscoelastic (II) fluid at successive times. 1) $\bar{t}$ = 0.05; 2) 0.25; 3) 0.5; 4) 1.75.

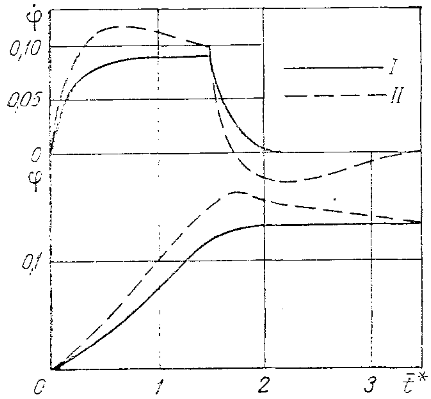

Fig. 3

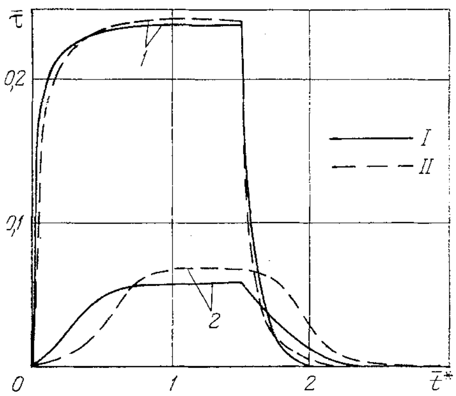

Fig. 4

Fig. 3. Angular velocity ($\dot{\varphi}$) and displacement (φ) of the inner cylinder rotating in a viscous (I) and a viscoelastic (II) fluid, $\bar{t}^*$ = 1.5.

Fig. 4. Evolution of the tangential stresses on the inner (1) and outer (2) cylinders for a viscous (I) and a viscoelastic (II) fluid, $\bar{t}^*$ = 1.5.

where $A = \sqrt{2\varepsilon/\lambda-((1 + 2\beta\varepsilon\lambda)/2\lambda)^2}$. As in the case $\varepsilon\lambda \gg 1$, the motion of the inner cylinder for $\varepsilon\lambda \sim (1 + \sqrt{1-\beta})^2/2\beta^2$ takes place with a velocity of rotation greater than the steady-state value for $t_\lambda \sim \beta\lambda/(1 + \sqrt{1-\beta}) \times \sqrt{1-\beta}$ (Fig. 1c). Steady rotation is established in a period $t \sim \beta\lambda/(1 + \sqrt{1-\beta}) < \lambda$. When $\varepsilon\lambda \sim (1 - \sqrt{1-\beta})^2/2\beta^2$, the velocity of rotation of the inner cylinder increases monotonically during a period $t \sim \beta\lambda/(1 - \sqrt{1-\beta}) > \lambda$.

Next we analyze numerical calculations of the problem in the complete formulation (in partial differential equations) described in [1]. Figure 2 characterizes the evolution of the velocity and tangential stress fields for a viscous (El = 0) and a viscoelastic (El = 1, $\alpha = 1.5$, $\varepsilon\lambda = 8.38$) liquid. Because of the time dependence of the effective viscosity for a viscoelastic fluid, its frictional resistance in the initial flow stage ($t \leqslant \lambda$) is lower than in a Newtonian fluid. In a viscoelastic fluid, therefore, the cylinder moves more rapidly than in a viscous fluid. The tangential stresses and the shear velocity near the inner cylinder are smaller in a viscoelastic fluid. It is seen in Fig. 3 that the velocity of rotation of the inner cylinder in a viscoelastic fluid arrives at a steady-state value by a monotonically increasing route. For a viscoelastic fluid the curve $\dot{\varphi}(t)$ in this stage of the process has a characteristic maximum and is situated above the corresponding curves for a viscous fluid. Because of the relaxation properties of the viscoelastic fluid the steady velocity of rotation of the cylinder is established for it later than for a purely viscous fluid. Inasmuch as the velocity of propagation of shear disturbances in a viscoelastic fluid is lower than in a viscous fluid (see the analysis of problem 1 in [1]), in the initial flow stage the region entrained by shear flow is smaller. Up to the time of removal of the external torque, $t = t^*$, the angle of rotation of the inner cylinder in a viscous fluid increases monotonically with time. In a viscoelastic fluid, owing to the greater velocity of rotation, its angular displacement is greater than the corresponding values for a viscous fluid.

After removal of the external torque the cylinder stops. In a viscous fluid its velocity of rotation decreases monotonically. In a viscoelastic fluid the removal of the driving torque results in "relief" of the elastic forces of the fluid. The cylinder therefore at first acquires momentum in the opposite direction (Fig. 3). Then its velocity decreases monotonically, the angle of rotation gradually decreasing and attaining the same value as in a viscous fluid. The coincidence of these quantities can be proved analytically for any type of relaxation functions. Because of the linearity of problem 3, the angle of rotation of the inner cylinder is related to the torque acting on it:

$$\varphi(t) = \int_0^t Q(t-t')\, dM(t'), \tag{6}$$

where Q(t) is the response function describing the variation of the angle of roation under the action of a constant unit torque applied at time $t = 0$ [$Q(t) = 0$ for $t \leqslant 0$]. For the situation in question the torque varies as follows:

$$M(t) = M \cdot H(t) - M \cdot H(t-t^*),$$

where H(t) is the Heaviside function, which is equal to unity for $t \geqslant 0$ and to zero for $t < 0$. Substituting the expression for M(t) into (6), we obtain

$$\varphi(t) = M[Q(t) - Q(t-t^*)]. \tag{7}$$

Let us examine the behavior of the function Q(t) as $t \to \infty$. It can be written in the form

$$Q(t) = \Omega_* t + \tilde{Q}(t), \tag{8}$$

where Ω_* is the steady angular velocity of rotation of the inner cylinder under the action of a constant unit driving torque, and the function $\tilde{Q}(t)$ describes the transient process, $\tilde{Q}(t) \to \text{const}$ as $t \to \infty$. Substituting (8) into (7), we have

$$\varphi_{st} = \lim_{t\to\infty} \varphi(t) = \Omega_{st} t^* + \lim_{t\to\infty} M[\tilde{Q}(t) - \tilde{Q}(t-t^*)] = \Omega_{st} t^*,$$

where $\Omega_{st} = M\Omega_*$. Since the elastic properties of the fluid do not affect the steady angular velocity of rotation, we have

$$\Omega_{st} = \frac{M\left[1 - \left(\frac{R_1}{R_2}\right)^2\right]}{4\pi\eta_0 R_1^2 L},$$

and the value of the angle through which the inner cylinder rotates depends only on the initial Newtonian viscosity η_0. The time variation of the tangential stresses on the inner cylinder for viscoelastic and viscous fluids differs only in the initial stage (Fig. 4), in which they are lower for a viscoelastic fluid. The calculations in problem 3 were carried out for N = 6.

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RELATION BETWEEN FLUID VISCOSITY AND COMPRESSIBILITY

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Values of the shear viscosity are computed with an error not exceeding the error of experiment by means of formulas deduced on the basis of the theory of a free volume.

Hydraulic systems with working fluid pressures of 30-120 MPa [1] are being used more and more in machine construction. It is known that the shear viscosity and density (compressiblity) of the working fluid depend on the temperature and pressure and exert significant influence on the characteristics and operability of hydraulic system mechanisms as they change, especially when using automatic control instruments [2].

The pressure dependences of the shear viscosity and density of the fluids are determined experimentally on unique specialized instruments and apparatus [3], which require a large expenditure of facilities and time. If an attempt is made to establish a connection between the shear viscosity and density (compressibility) of the fluid, the expenditure in laboratory investigations could be reduced considerably.

Bachinskii [4] proposed the first empirical dependence between the shear viscosity and specific volume (density)

$$\eta = \frac{C}{V - \omega}. \tag{1}$$

Experimental confirmation of the formula yielded satisfactory agreement for a large number of fluids. However, for fluids associated by hydrogen bonds (alcohols, acids, water), some hydrogen halides, and mercury, the formula (1) turned out to be unacceptable. It yields such false results at high pressures. Nevertheless, investigations tracing the connection between the shear viscosity and density (compressibility) of a fluid continued. Thus, the authors of [5] found a relationship between these quantities for individual paraffins, the dependence $\eta = f(\rho)$ was derived in [6] for polysiloxanes, and an attempt was made in [7, 8] to establish such a relation for commercial mineral oils. We try to establish these relationships in an example of two working fluids of a hydraulic system that differ substantially in their chemical nature.

The uninflammable fluid PGV (on a water—glycerine base) is being more and more extensively used at this time in hydraulic systems with elevated fire-safety requirements instead of the shaft oil AU. Properties of the fluids PGV and shaft oil AU are compared in [9].

The pressure dependences of the viscosity of the fluid PGV and the shaft oil AU were determined on a viscosimeter with a rolling ball with a ±2-8% error [10], and the density (compressibility) by a hydrostatic method with ±0.2% error [9, 11].

An equation of state of the fluid (Tait equation) [12] was obtained from the experimental dependence of the compressibility

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